$\mathcal{U}$'s i-th tape in position x at time t when the tuple Z designates the initial configuration on $\mathcal{U}$'s tapes.

(2) $\text{Head}_i(Z, t)$ indicating the address of the i-th tape's head at time t when the tuple Z designates the initial configuration on $\mathcal{U}$'s tapes.

(3) $\text{State}_i(Z, t) = 1$ if the machine $\mathcal{U}$ is in state i at time t when the tuple Z designates the initial configuration on $\mathcal{U}$'s tapes. $\text{State}_i(Z, t) = 0$ otherwise.

Given a fixed constant λ and an arbitrary initial axiom system A, $\text{ISTM}(A)$ is defined to include the following three groups of axioms:

Group-1. An initial finite set of Π_1 axioms which formally define the properties of $\text{ISTM}(A)$'s base functions and of the two relations $=$ and $<$.

Group-2. An infinite axiom schema which for each Π_1 sentence Φ, asserts if a proof of Φ from A exists then Φ.

Group-3. For arbitrary x and arbitrary tuple Z, the assertion that the machine $\mathcal{U}$ is unable to produce a proof from $\text{ISTM}(A)$ of $0 = 1$ on any of its tapes when it runs for x^λ units of time.

Some of our main theorems about $\text{ISTM}(A)$ are listed below:

(1) If the union of A with Group-1 (above) is consistent then $\text{ISTM}(A)$ is consitent.

(2) The above theorem is not true for all A when $\text{ISTM}(A)$'s Group-1 axioms are merely expanded to incorporate Addition as a total function.

(3) $\text{ISTM}(A)$ may use a version of deduction that permits Gentzen-style cuts provided the Successor function is removed from Group-1. (Added Comment: Robert Solovay [1] has proven a theorem which implies that $\text{ISTM}(A)$, with Gentzen-style cuts, can be inconsistent if Successor is not removed.)

(4) The several techniques, itemized in our preceding papers [2, 3, 4], to strengthen $\text{IS}(A)$'s Group-3 axiom schema are also applicable for strenghtenning $\text{ISTM}(A)$. For example, Group-3 could state that if for some Z, x and Σ_1 formula Ψ, $\mathcal{U}$ can construct in time $O(x^\lambda)$ a proof from $\text{ISTM}(A)$ of $\forall x\, \Psi(x)$ then $\Psi(\bar{c})$ is valid for each fixed $\bar{c}$

[1] R. Solovay, private communications, April 1994.

[2] D. Willard, *Self-verifying axiom systems*, ***Proceedings of the Third Kurt Gödel Symposium***, 1993, pp. 325–336, published in Springer-Verlag Lecture Notes in Computer Science, vol. 713.

[3] ———, *Self-verifying axiom systems and the incompleteness theorem*, talk presented at the 1994 Annual Meeting of the ASL, summary of talk in this Journal (1995), pp. 136–137, the journal version of this article available as University of Albany TR 93-10 and to appear in ***Journal of Symbolic Logic***.

[4] ———, *Stronger variants of self-verifying axiom systems*, talk presented at the Association for Symbolic Logic's 1995 Winter Meeting, summary of talk available in this Journal (1995), pp. 371–372.

Abstracts presented by title

► STANLEY BURRIS and ANDRÁS SÁRKÖZY, *Fine spectra and first-order limit laws*.

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Using Feferman-Vaught techniques we show a certain property of the fine spectrum of an admissible class of structures leads to a first-order law. The condition presented is best possible in the sense that if it is violated then one can find an admissible class with the same fine spectrum which does not have a first-order law.

Our first result for verifying that the above condition actually holds is to show that the count function of an admissible class has regular variation with a certain uniformity of convergence. This condition applies to a wide range of admissible classes, including those satisfying Knopfmacher's Axiom A, and those satisfying Bateman and Diamond's condition.

The second result applies to the case that the sizes of the structures in an admissible class K are all powers of a single integer, and there is a uniform bound on the number of K-indecomposables of any given size.

▶ D. A.BREDIKHIN, *Graphs calculus and quasieqational theories of relation algebras*.
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Let **K** be the class of algebras which is isomorphic to ones whose elements are binary relations and whose operations are the following: relation product $\circ$, relation inverse $^{-1}$, intersection $\cap$ and the diagonal relation Δ. This class was introduced by B. Jónsson [1]. The equational theory of **K** was characterized in [2,3]. Here the quasiequational theory of **K** shall be described.

Graphs Calculus. A (2-pointed labeled) graph is a structure $G = (V, E, i, o)$ where V is a set, $i, o \in V$ and $E \subset V \times N \times V$ $(N = 1, 2, \dots)$. We write that $G_1 \prec G_2$ if there exists an homomorphism from G_2 to G_1. Denote by $G_1[u, v, G_2]$ a graph which is obtained from the disjoined union of G_1 and G_2 by identifying i_2 with u and o_2 with v.

Let us consider a formal system $\mathcal{L}$ determined as follows. Formulae of $\mathcal{L}$ are expressions of the form $\varphi = G_1 \Rightarrow G_2$ where G_1 and G_2 are graphs. Axioms: $G \Rightarrow G$. Rules of inference:

$$\frac{G_1 \Rightarrow G_2, G_2 \Rightarrow G_3}{G_1 \Rightarrow G_3}, \quad \frac{G_1 \Rightarrow G_2, G_2 \prec G_3}{G_1 \Rightarrow G_3}, \quad \frac{G_1 \Rightarrow G_2}{G[u, v, G_1] \Rightarrow G[u, v, G_2]}.$$

We write that $\varphi_1, \dots, \varphi_n \vdash \varphi_0$ if φ_0 deduce from $\varphi_1, \dots, \varphi_n$.

Let $(A, \cdot, ^{-1}, \wedge, 1)$ be an algebra of the type $(2, 1, 2, 0)$. With each its term p can be associated a graph $G(p)$ (see in [2,3]).

THEOREM 1. *The quasi-identity* $\bigwedge_{k=1}^{n} p_k = q_k \rightarrow p_0 = q_0$ *belongs to the quasiequational theory of* **K** *if and only if* $\varphi_1, \dots, \varphi_n \vdash \varphi_0$ *and* $\tilde{\varphi}_1, \dots, \tilde{\varphi}_n \vdash \tilde{\varphi}_0$ *where* $\varphi_k = G(p_k) \Rightarrow G(q_k)$ *and* $\tilde{\varphi}_k = G(q_k) \Rightarrow G(p_k)$.

This result can be generalized to classes of algebras of relations with primitive-positive operations.

[1] B. JÓNSSON, *Representation of modular lattices and relation algebras*, ***Transactions of the American Mathematical Society***, vol. 92 (1959), pp. 449–464.

[2] H. ANDRÉKA and D. A. BREDIKHIN, *The equational theory of union-free algebras of relations*, ***Algebra Universales***, to appear.

[3] D. A. BREDIKHIN, *The equational theory of algebras of relations with positive operations*, ***Izvestia Vysshikh Uchebnykh Zavedenii, Matematicheskaya***, vol. 3 (1993), pp. 23-30, Russian.

▶ ZINOVY DISKIN, *Towards graph-based algebraic model theory for computer science*.
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For theoretical support of modern information technologies Computer Science needs a model theory framework for studying structures over graphs whose nodes are sets and edges are functions. A suitable concept was developed in category theory under the name of *sketches*. However, while the familiar notion presupposes a few fixed diagram properties (commutativity, (co)limitness), the applications require the possibility to set arbitrary vocabularies of diagram properties.

DEFINITION 1. (i) *A (finitary) predicate signature* is a set π of symbols called *markers* s.t. each $m \in \pi$ is ssigned with a (finite) graph D_m called the *arity shape* of m.

A π-sketch is a couple $S = (G, \mathcal{D})$ where G is a graph and $\mathcal{D} = (\mathcal{D}_m, m \in \pi)$ is a family with $\mathcal{D}_m \subset \mathbf{Hom}(D_m, G)$ a collection (maybe, empty) of diagrams *marked* by m. *A morphism* of π-sketches $S \to S'$ is a graph morphism $h\colon\ G \to G'$ s.t. $\delta; h \in \mathcal{D}'_m$ as soon as $\delta \in \mathcal{D}_m$.

(ii) *An operation signature over* π is a set φ of *operation markers* s.t. each $q \in \varphi$ is assigned with a π-sketch inclusion $D_q^{\text{in}} \subset D_q^{\text{out}}$ with D_q^{out} called the *output shape* of the operation and D_q^{in} *the input* one. An operation is called *finitary* if its output shape is finite.

A π-sketch A is said to be *φ-algebra* if for any $q \in \varphi$ there is defined a mapping

$$q^A\colon\ \mathbf{Hom}_\pi(D_q^{\text{in}}, A) \to \mathbf{Hom}_\pi(D_q^{\text{out}}, A)$$

s.t. $q^A(\mu)\lceil_{D_q^{\text{in}}} = \mu$ for any morphism $\mu\colon\ D_q^{\text{in}} \to A$.

Since an operation marker is simultaneously a predicate marker of arity D_q^{out}, the construction can be iterated. Actually, a *signature* σ of markers should be a partly ordered set s.t. the arity sketch of a marker q can involve only markers preceding q.

Starting from these basic definitions, a conceptual framework for graph-based model theory is proposed in the paper. The main technical contribution is the accurate development of the machinery of diagram operations over generalized sketches defined above. Specifically, constructions of composing such operations and of the free algebra generated by a sketch were elaborated.

Given a signature σ of predicate and operation markers, a notion of *σ-tree* is defined in a suitable way. Very roughly, this is a tree whose nodes are labeled by pairs (q, δ) with δ a diagram $\delta\colon\ D_q^{\text{out}} \to V$ and V a fixed pool of variables. With any σ-tree T there are correlated (i) a corresponding σ-sketch $\text{Ske}(T)$ obtained by merging all diagrams and (ii) its subsketch $\text{In}(T)$ obtained by merging all diagrams in the leaves.

DEFINITION 2. A *σ-specification* is a pair $\mathbf{S} = (S, X)$ with S a finitary σ-sketch and X a distinguished subobject of its carrier called *the input object* of $\mathbf{S}$. $\mathbf{S}$ is called *well-formed* (wf) if there is a σ-tree T s.t. $S = \text{Ske}(T)$ and $X = \text{In}(T)$.

PROPOSITION 3 (Sketch parsing). *There is an algorithm either extracting from any given specification* $\mathbf{S} = (S, X)$ *a tree* $T = \text{Tree}(\mathbf{S})$ *s.t.* $S = \text{Ske}(T)$ *and* $X = \text{In}(T)$*, or terminated with failure proving that* $\boldsymbol{S}$ *is not wf. So, the question of whether a given σ-specification is well-formed is decidable.*

► M. Z. GRULOVIĆ and M. S. KURILIĆ, *Some results on reduced ideal-products*.
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In [4] it is proved that a reduced ideal-product $\prod_\Psi^\Lambda \mathcal{X}_i$ (Λ and Ψ being, respectively, an ideal and a filter on the index set I) preserves the separation axioms $T_k,\ k = 0, 1, 2, 3, 3\frac{1}{2}$, iff the following, named $(\Lambda\Psi)$-condition, holds: $\forall A \in \Psi\, \forall B \notin \Psi\, \exists L \in \Lambda\, (L \subseteq A \cap B^c \wedge L^c \notin \Psi)$, which is, on the other side, equivalent to the condition that the set $(\Lambda/\equiv) \setminus \{\mathbf{0}\}$ is dense in $(P(I)/\equiv) \setminus \{\mathbf{0}\}$, where, for $A, B \in P(I),\ A \equiv B$ iff $\exists F \in \Psi\ A \cap F = B \cap F$.